Natural Language Processing

Language modeling

Yulia Tsvetkov

yuliats@cs.washington.edu

PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

Learning components

A loss function:

- **cross-entropy loss**

$$L_{\mathrm{CE}}(\hat{y}, y) = -[y \log \sigma(\mathbf{w} \cdot \mathbf{x} + b) + (1-y) \log(1 - \sigma(\mathbf{w} \cdot \mathbf{x} + b))]$$

An optimization algorithm:

- **stochastic gradient descent**

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} L_{\mathrm{CE}}(f(x^{(i)}; \theta), y^{(i)})$$

```
function STOCHASTIC GRADIENT DESCENT($L()$, $f()$, $x$, $y$) returns $\theta$
    # where: L is the loss function
    #        f is a function parameterized by $\theta$
    #        x is the set of training inputs $x^{(1)}, x^{(2)}, ..., x^{(m)}$
    #        y is the set of training outputs (labels) $y^{(1)}, y^{(2)}, ..., y^{(m)}$

$\theta \leftarrow 0$
repeat til done
  For each training tuple $(x^{(i)}, y^{(i)})$ (in random order)
    1. Optional (for reporting):              # How are we doing on this tuple?
       Compute $\hat{y}^{(i)} = f(x^{(i)};\theta)$             # What is our estimated output $\hat{y}$?
       Compute the loss $L(\hat{y}^{(i)}, y^{(i)})$    # How far off is $\hat{y}^{(i)}$) from the true output $y^{(i)}$?
    2. $g \leftarrow \nabla_\theta L(f(x^{(i)};\theta), y^{(i)})$       # How should we move $\theta$ to maximize loss?
    3. $\theta \leftarrow \theta - \eta g$                    # Go the other way instead
return $\theta$
```

Hyperparameters

The learning rate η is a **hyperparameter**

- too high: the learner will take big steps and overshoot
- too low: the learner will take too long

Hyperparameters:

- Briefly, a special kind of parameter for an ML model
- Instead of being learned by algorithm from supervision (like regular parameters), they are chosen by algorithm designer.

Mini-batch training

Stochastic gradient descent chooses a single random example at a time.

That can result in choppy movements

More common to compute gradient over batches of training instances.

Batch training: entire dataset

Mini-batch training: m examples (512, or 1024)

Overfitting

A model that perfectly match the training data has a problem.

It will also **overfit** to the data, modeling noise

- A random word that perfectly predicts y (it happens to only occur in one class) will get a very high weight.
- Failing to generalize to a test set without this word.

A good model should be able to **generalize**

Regularization

A solution for overfitting

Add a **regularization** term R(θ) to the loss function (for now written as maximizing logprob rather than minimizing loss)

$$\hat{\theta} \;=\; \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{m} \log P(y^{(i)}|x^{(i)}) - \alpha R(\theta)$$

Idea: choose an R(θ) that penalizes large weights

- fitting the data well with lots of big weights not as good as fitting the data a little less well, with small weights

L2 regularization (ridge regression)

The sum of the squares of the weights

$$R(\theta) = ||\theta||_2^2 = \sum_{j=1}^{n} \theta_j^2$$

L2 regularized objective function:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \left[\sum_{i=1}^{m} \log P(y^{(i)}|x^{(i)}) \right] - \alpha \sum_{j=1}^{n} \theta_j^2$$

L1 regularization (=lasso regression)

The sum of the (absolute value of the) weights

$$R(\theta) = ||\theta||_1 = \sum_{i=1}^{n} |\theta_i|$$

L1 regularized objective function:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \left[\sum_{1=i}^{m} \log P(y^{(i)}|x^{(i)}) \right] - \alpha \sum_{j=1}^{n} |\theta_j|$$

Multinomial Logistic Regression

Often we need more than 2 classes

- Positive/negative/neutral
- Parts of speech (noun, verb, adjective, adverb, preposition, etc.)
- Classify emergency SMSs into different actionable classes

If >2 classes we use **multinomial logistic regression**

= Softmax regression

= Multinomial logit

= (defunct names : Maximum entropy modeling or MaxEnt

So "logistic regression" will just mean binary (2 output classes)

Multinomial Logistic Regression

The probability of everything must still sum to 1

P(positive|doc) + P(negative|doc) + P(neutral|doc) = 1

Need a generalization of the sigmoid called the **softmax**

- Takes a vector $z = [z_1, z_2, ..., z_k]$ of k arbitrary values
- Outputs a probability distribution
- each value in the range $[0,1]$
- all the values summing to 1

We'll discuss it more when we talk about neural networks

Components of a probabilistic machine learning classifier

Given m input/output pairs $(x^{(i)}, y^{(i)})$:

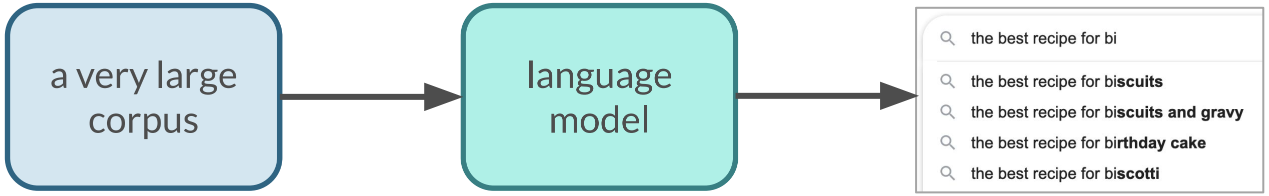
1. A **feature representation** for the input. For each input observation $x^{(i)}$, a vector of features $[x_1, x_2, \ldots, x_n]$. Feature j for input $x^{(i)}$ is x_j, more completely $x_1^{(i)}$, or sometimes $f_j(x)$.

2. A **classification function** that computes $\hat{y}$ the estimated class, via $p(y|x)$, like the **sigmoid** or **softmax** functions

3. An **objective function** for learning, like **cross-entropy loss**

4. An algorithm for **optimizing** the objective function: **stochastic gradient descent**

Language modeling



the best recipe for bi

the best recipe for **biscuits**

the best recipe for **biscuits and gravy**

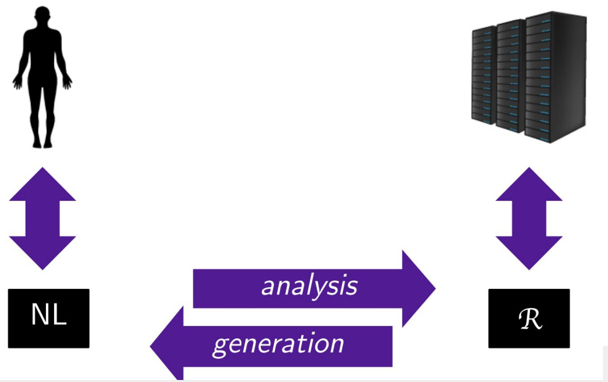
the best recipe for **birthday cake**

the best recipe for **biscotti**

What is Natural Language Processing (NLP)?

- NL ∈ {Mandarin Chinese, Hindi, Spanish, Arabic, English, … Inuktitut, Njerep}

- Automation of NLs:
  - analysis of ("understanding") what a text means, to some extent (NL → $\mathcal{R}$)
  - generation of fluent, meaningful, context-appropriate text ($\mathcal{R}$ → NL)
  - acquisition of $\mathcal{R}$ from knowledge and data



elijah wood
everything is illuminated

My legal name is Alexander Perchov.

My legal name is Alexander Perchov. But all of my many friends dub me Alex, because that is a more flaccid-to-utter version of my legal name.

*My legal name is Alexander Perchov. But all of my many friends dub me Alex, because that is a more flaccid-to-utter version of my legal name. Mother dubs me Alexi-stop-spleening-me!, because I am always
spleening her.*

My legal name is Alexander Perchov. But all of my many friends dub me Alex, because that is a more flaccid-to-utter version of my legal name. Mother dubs me Alexi-stop-spleening-me!, because I am always
spleening her. If you want to know why I am always spleening her, it is because I am always elsewhere with friends, and disseminating so much currency, and performing so many things that can spleen a mother.

My legal name is Alexander Perchov. But all of my many friends dub me Alex, because that is a more flaccid-to-utter version of my legal name. Mother dubs me Alexi-stop-spleening-me!, because I am always
spleening her. If you want to know why I am always spleening her, it is because I am always elsewhere with friends, and disseminating so much currency, and performing so many things that can spleen a mother. Father used to dub me Shapka, for the fur hat I would don even in the summer month.

My legal name is Alexander Perchov. But all of my many friends dub me Alex, because that is a more flaccid-to-utter version of my legal name. Mother dubs me Alexi-stop-spleening-me!, because I am always
spleening her. If you want to know why I am always spleening her, it is because I am always elsewhere with friends, and disseminating so much currency, and performing so many things that can spleen a mother. Father used to dub me Shapka, for the fur hat I would don even in the summer month. He ceased dubbing me that because I ordered him to cease dubbing me that. It sounded boyish to me, and I have always thought of myself as very potent and generative.

Language models play the role of ...

- a judge of grammaticality
- a judge of semantic plausibility
- an enforcer of stylistic consistency
- a repository of knowledge (?)

The Language Modeling problem

- Assign a probability to every sentence (or any string of words)
  - finite vocabulary (e.g. words or characters) *{the, a, telescope, …}*
  - infinite set of sequences
    - *a telescope STOP*
    - *a STOP*
    - *the the the STOP*
    - *I saw a woman with a telescope STOP*
    - *STOP*
    - *...*

The Language Modeling problem

- Assign a probability to every sentence (or any string of words)
  - finite vocabulary (e.g. words or characters)
  - infinite set of sequences

$$\sum_{\mathbf{e}\in\Sigma^*} p_{\text{LM}}(\mathbf{e}) = 1$$

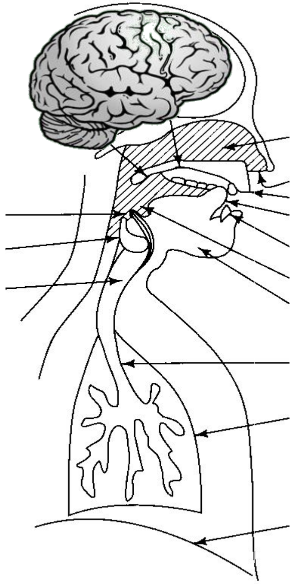
$$p_{\text{LM}}(\mathbf{e}) \geq 0 \quad \forall \mathbf{e} \in \Sigma^*$$

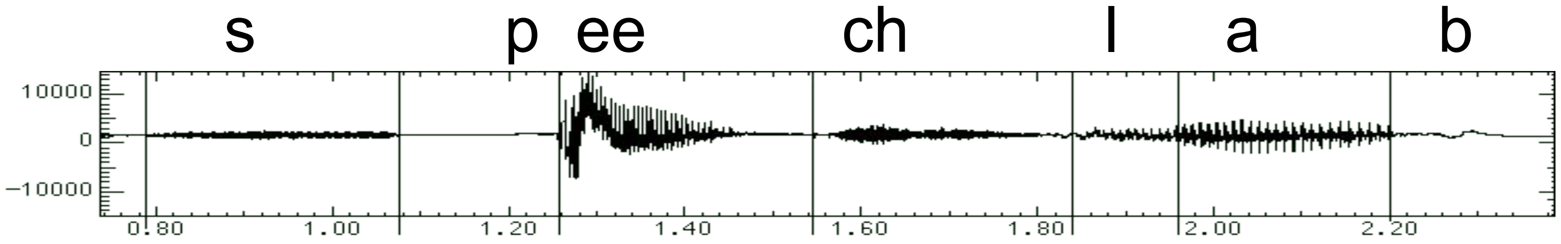
p(*disseminating so much currency STOP*) = 10^{-15}

p(*spending a lot of money STOP*) = 10^{-9}

Motivation

- Speech recognition: we want to predict a sentence given acoustics





Motivation

- Speech recognition: we want to predict a sentence given acoustics

the station signs are indeed in english	-14725
the station signs are in deep in english	-14732
the stations signs are in deep in english	-14735
the station signs are in deep into english	-14739
the station 's signs are in deep in english	-14740
the station signs are in deep in the english	-14741
the station 's signs are indeed in english	-14760
the station signs are indians in english	-14790
the station signs are indian in english	-14799
the stations signs are indians in english	-14807
the stations signs are indians and english	-14815

Motivation

- Machine translation
  - p(*strong winds*) > p(*large winds*)
- Spelling correction
  - The office is about fifteen minuets from my house
  - p(*about fifteen minutes from*) > p(*about fifteen minuets from*)
- Speech recognition
  - p(*I saw a van*) >> p(*eyes awe of an*)
- Summarization, question-answering, handwriting recognition, OCR, etc.

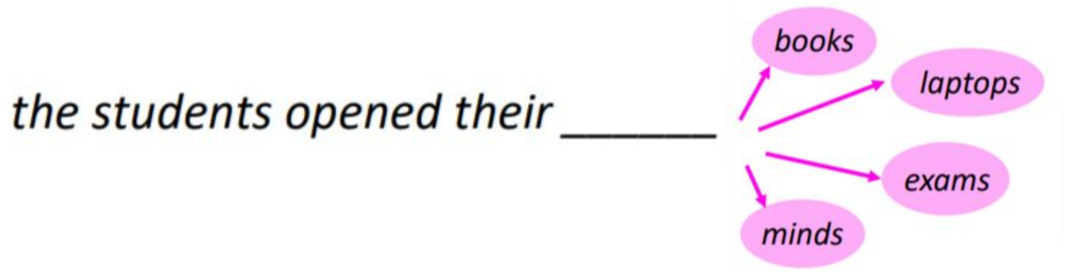
Equivalent definition

- **Language Modeling** is the task of predicting what word comes next

the students opened their ______

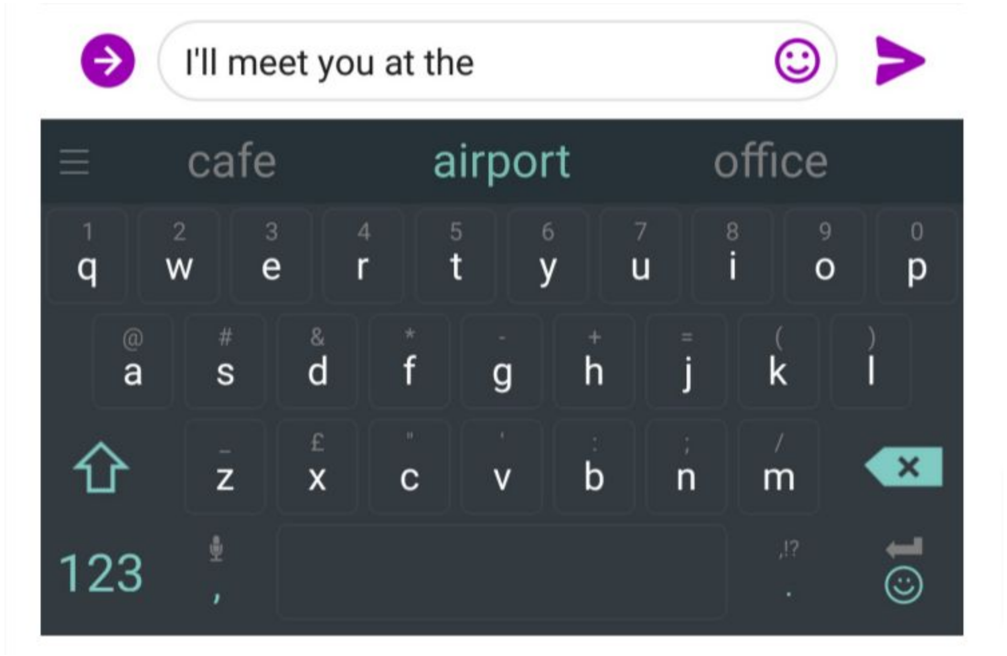
Equivalent definition

- **Language Modeling** is the task of predicting what word comes next



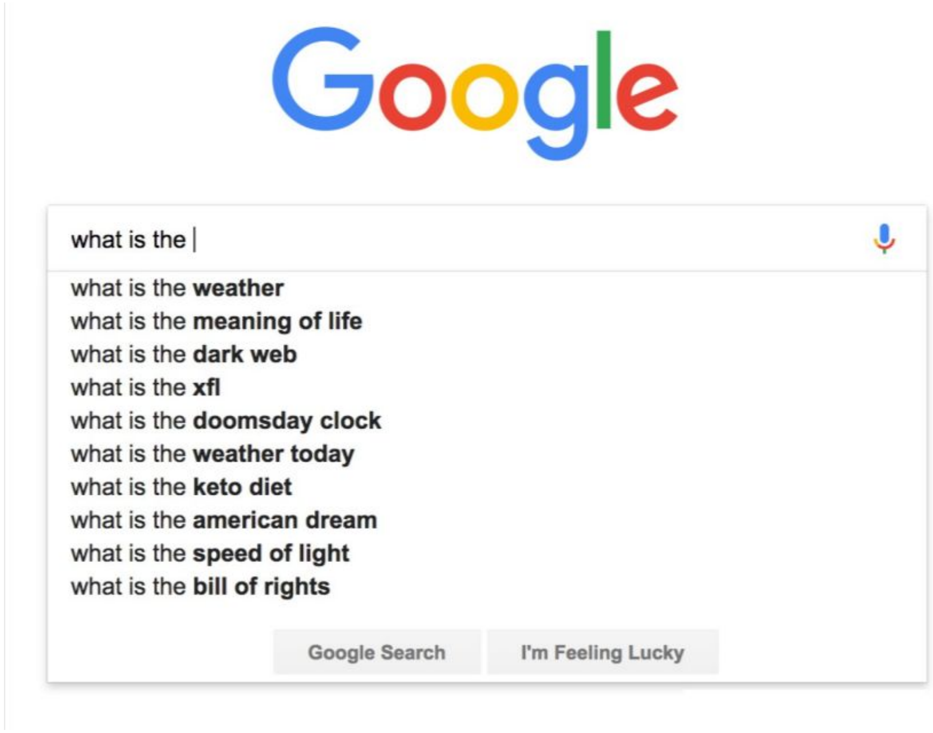
- More formally: given a sequence of words $x^{(1)}, x^{(2)}, \dots x^{(t)}$
  compute the probability distribution if the next word $x^{(t+1)}$
  Where $x^{(t+1)}$ can be any word in the vocabulary $V=\{ w_1, w_2, \dots w_{|V|}\}$

We use Language Models every day


I'll meet you at the

cafe airport office

We use Language Models every day



what is the |

what is the **weather**
what is the **meaning of life**
what is the **dark web**
what is the **xfl**
what is the **doomsday clock**
what is the **weather today**
what is the **keto diet**
what is the **american dream**
what is the **speed of light**
what is the **bill of rights**

Language Modeling

- If we have some text, then the probability of this text (according to the Language Model) is:

$$\begin{aligned} P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) &= P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \, \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \, \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)}) \\ &= \prod_{t=1}^{T} \underbrace{P(\boldsymbol{x}^{(t)} | \, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})}_{\text{This is what our LM provides}} \end{aligned}$$

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Question: How to learn a Language Model?

A trivial model

- Assume we have n training sentences
- Let $x_1, x_2, \ldots, x_n$ be a sentence, and $c(x_1, x_2, \ldots, x_n)$ be the number of times it appeared in the training data.
- Define a language model:

$$p(x_1, \ldots, x_n) = \frac{c(x_1, \ldots, x_n)}{N}$$

- No generalization!

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Question: How to learn a Language Model?
- Answer (pre- Deep Learning): learn an *n-gram* Language Model!

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Definition: An n-gram is a chunk of n consecutive words.

n-gram Language Models

“I have a dog whose name is Lucy. I have two cats, they like playing with Lucy.”

- Definition: An n-gram is a chunk of n consecutive words.
  - unigrams: {I, have, a, dog, whose, name, is, Lucy, two, cats, they, like, playing, with}

n-gram Language Models

“I have a dog whose name is Lucy. I have two cats, they like playing with Lucy.”

- Definition: An n-gram is a chunk of n consecutive words.
  - unigrams: {I, have, a, dog, whose, name, is, Lucy, two, cats, they, like, playing, with}
  - bigrams: {I have, have a, a dog, dog whose, … , with Lucy}

n-gram Language Models

“I have a dog whose name is Lucy. I have two cats, they like playing with Lucy.”

- Definition: An n-gram is a chunk of n consecutive words.
  - unigrams: {I, have, a, dog, whose, name, is, Lucy, two, cats, they, like, playing, with}
  - bigrams: {I have, have a, a dog, dog whose, … , with Lucy}   have cats

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Definition: An n-gram is a chunk of n consecutive words.
  - unigrams: {I, have, a, dog, whose, name, is, Lucy, two, cats, they, like, playing, with}
  - bigrams: {I have, have a, a dog, dog whose, … , with Lucy}   have ✗ cats

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Definition: An n-gram is a chunk of n consecutive words.
  - unigrams: {I, have, a, dog, whose, name, is, Lucy, two, cats, they, like, playing, with}
  - bigrams: {I have, have a, a dog, dog whose, … , with Lucy}
  - trigrams: {I have a, have a dog, a dog whose, … , playing with Lucy}

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Definition: An n-gram is a chunk of n consecutive words.
  - unigrams: {I, have, a, dog, whose, name, is, Lucy, two, cats, they, like, playing, with}
  - bigrams: {I have, have a, a dog, dog whose, … , with Lucy}
  - trigrams: {I have a, have a dog, a dog whose, … , playing with Lucy}
  - four-grams: {I have a dog, … , like playing with Lucy}
  - …

n-gram Language Models

“I have a dog whose name is Lucy. I have two cats, they like playing with Lucy.”

- w_1 – a unigram
- $w_1\ w_2$ – a bigram
- $w_1\ w_2\ w_3$ – a trigram
- $w_1 w_2 \ldots w_n$ – an n-gram

n-gram Language Models

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

- Question: How to learn a Language Model?
- Answer (pre- Deep Learning): learn an *n-gram* Language Model!

- Idea: Collect statistics about how frequent different n-grams are and use these to predict next word

unigram probability

“I have a dog whose name is Lucy. I have two cats, they like playing with Lucy.”

- corpus size m = 17
- P(Lucy) = 2/17; P(cats) = 1/17

- Unigram probability: $P(w) = \frac{\text{count}(w)}{m} = \frac{C(w)}{m}$

bigram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$P(\text{have} \mid \text{I}) = \frac{P(\text{I have})}{P(\text{I})} = \frac{2}{2} = 1$$

$$P(\text{two} \mid \text{have}) = \frac{P(\text{have two})}{P(\text{have})} = \frac{1}{2} = 0.5$$

$$P(\text{eating} \mid \text{have}) = \frac{P(\text{have eating})}{P(\text{have})} = \frac{0}{2} = 0$$

$$P(w_2|w_1) = \frac{C(w_1,w_2)}{\sum_w C(w_1,w)} = \frac{C(w_1,w_2)}{C(w_1)}$$

trigram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$P(\text{a} \mid \text{I have}) = \frac{C(\text{I have a})}{C(\text{I have})} = \frac{1}{2} = 0.5$$

$$P(w_3 | w_1\, w_2) = \frac{C(w_1,w_2,w_3)}{\sum_w C(w_1,w_2,w)} = \frac{C(w_1,w_2,w_3)}{C(w_1,w_2)}$$

$$P(\text{several} \mid \text{I have}) = \frac{C(\text{I have several})}{C(\text{I have})} = \frac{0}{2} = 0$$

n-gram probability

"I have a dog whose name is Lucy. I have two cats, they like playing with Lucy."

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$

$$P(w_i \mid w_1, w_2, \ldots, w_{i-1}) = \frac{C(w_1, w_2, \ldots, w_{i-1}, w_i)}{C(w_1, w_2, \ldots, w_{i-1})}$$

Sentence/paragraph/book probability

$$\begin{aligned} P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) &= P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \, \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \, \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)}) \\ &= \prod_{t=1}^{T} P(\boldsymbol{x}^{(t)} | \, \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)}) \end{aligned}$$

P(its water is so transparent that the) =

P(its) ×
P(water | its) ×
P(is | its water) ×
P(so | its water is) ×
P(transparent | its water is so) ×
… ×

P(the | its water is so transparent that) → How to estimate?

Markov assumption

- We make the Markov assumption: $x^{(t+1)}$ depends only on the preceding n-1 words
  - Markov chain is a "*…stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.*"

Andrei Markov

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\underbrace{\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)}}_{\text{n-1 words}})$$

assumption

Markov assumption

Andrei Markov

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

or maybe even

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

First-order Markov process

Chain rule

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$

$$p(X_1 = x_1) \prod_{i=2}^{n} p(X_i = x_i \mid X_1 = x_1, \ldots, X_{i-1} = x_{i-1})$$

First-order Markov process

Chain rule

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$

$$p(X_1 = x_1) \prod_{i=2}^{n} p(X_i = x_i \mid X_1 = x_1, \ldots, X_{i-1} = x_{i-1})$$

Markov assumption

$$= \quad P(X_1 = x_1) \prod_{i=2}^{n} P(X_i = x_i | X_{i-1} = x_{i-1})$$

Second-order Markov process:

- Relax independence assumption:

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$
$$p(X_1 = x_1) \times p(X_2 = x_2 \mid X_1 = x_1)$$
$$\times \prod_{i=3}^{n} p(X_i = x_i \mid X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1})$$

Second-order Markov process:

- Relax independence assumption:

$$p(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) =$$
$$p(X_1 = x_1) \times p(X_2 = x_2 \mid X_1 = x_1)$$
$$\times \prod_{i=3}^{n} p(X_i = x_i \mid X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1})$$

- Simplify notation:

$$x_0 = *, x_{-1} = *$$

3-gram LMs

- A trigram language model contains
  - a vocabulary V
  - a non negative parameters q(*w*|*u*,*v*) for every trigram, such that

$$w \in \mathcal{V} \cup \{\text{STOP}\},\ \ u, v \in \mathcal{V} \cup \{*\}$$

  - the probability of a sentence x_1, …, x_n, where x_n=STOP is

$$p(x_1, \ldots, x_n) = \prod_{i=1}^{n} q(x_i \mid x_{i-1}, x_{i-2})$$

Example

$$p(\text{the dog barks STOP}) =$$

Example

$$p(\text{the dog barks STOP}) = q(the \mid *, *) \times$$

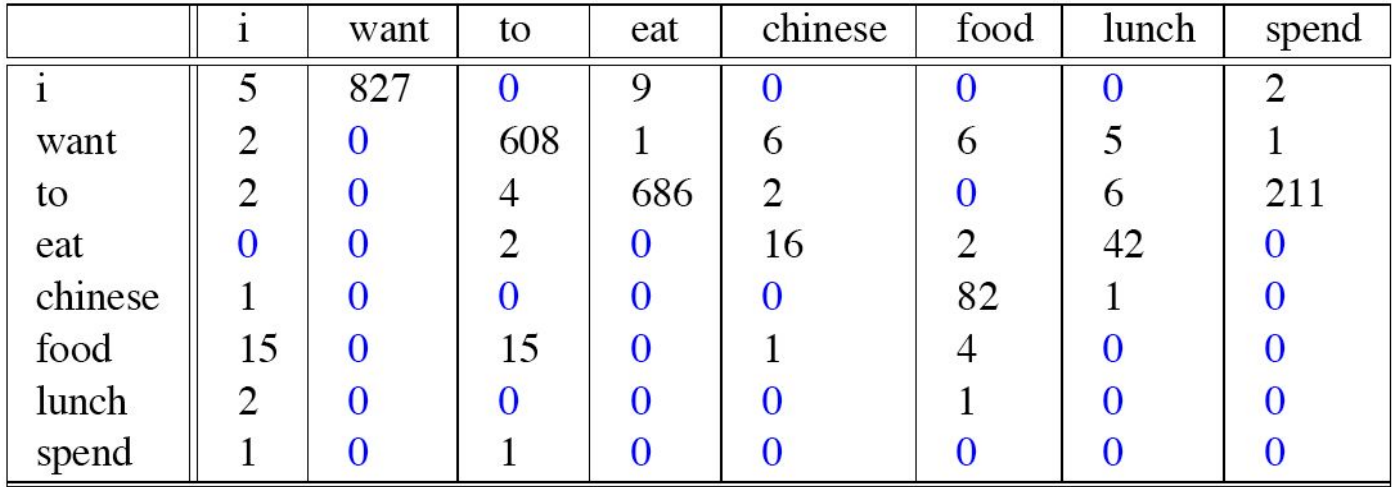
Example

$$\begin{aligned} p(\text{the dog barks STOP}) = & q(the \mid *, *) \times \\ & q(dog \mid *, the) \times \\ & q(barks \mid the, dog) \times \\ & q(STOP \mid dog, barks) \times \end{aligned}$$

Berkeley restaurant project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced that food is what i’m looking for
- tell me about chez pansies
- can you give me a listing of the kinds of food that are available
- i’m looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts (~1000 sentences)



	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Bigram probabilities

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

$$P(w_1, w_2, \ldots, w_n) \approxeq \prod_i P(w_i \mid w_{i-1})$$

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

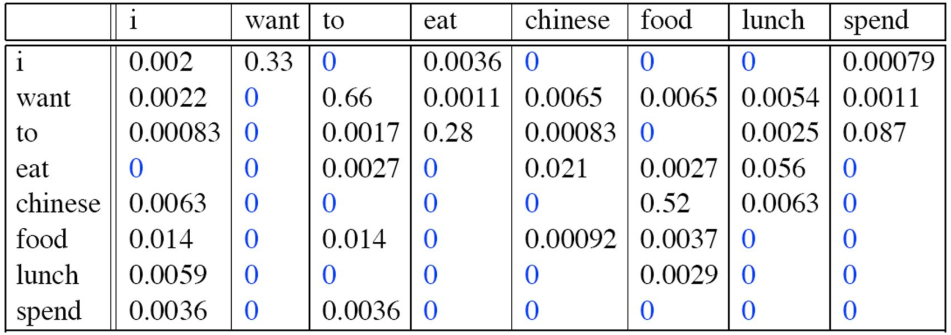
	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probability

$$P(w_i \mid w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}$$

$$P(w_1, w_2, \ldots, w_n) \approx \prod_i P(w_i \mid w_{i-1})$$

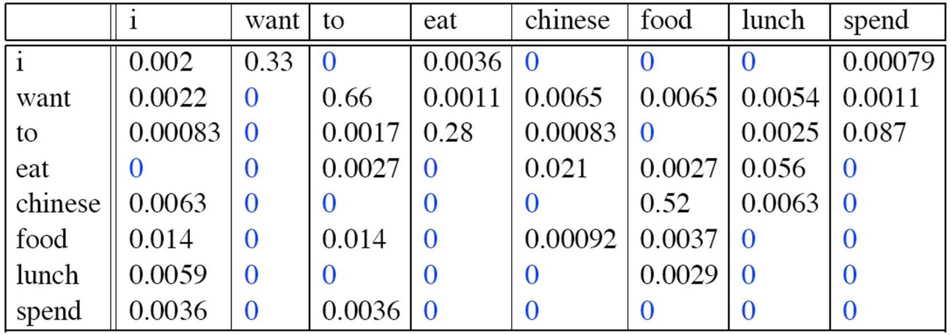
```
P(<s> i want chinese food </s>) =
P(i|<s>)
×   P(want|i)
×   P(chinese|want)
×   P(food|chinese)
×   P(</s>|food)
=   ...
```



	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

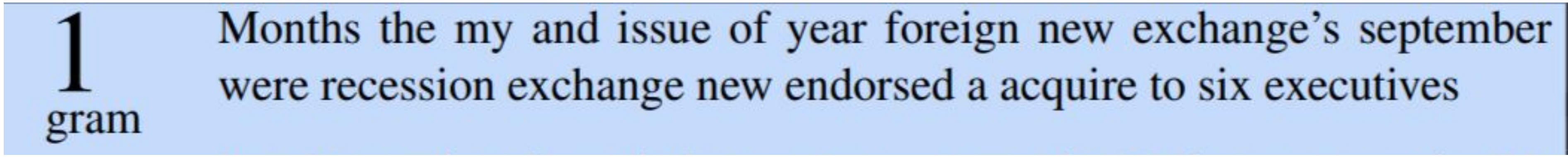
What can we learn from bigram estimates?

```
P(to|want)      = 0.66
P(chinese|want)    = 0.0065
P(eat|to)       = 0.28
P (i|<s>)       = 0.25
P(food|to)      = 0.0
P(want|spend)   = 0.0
```



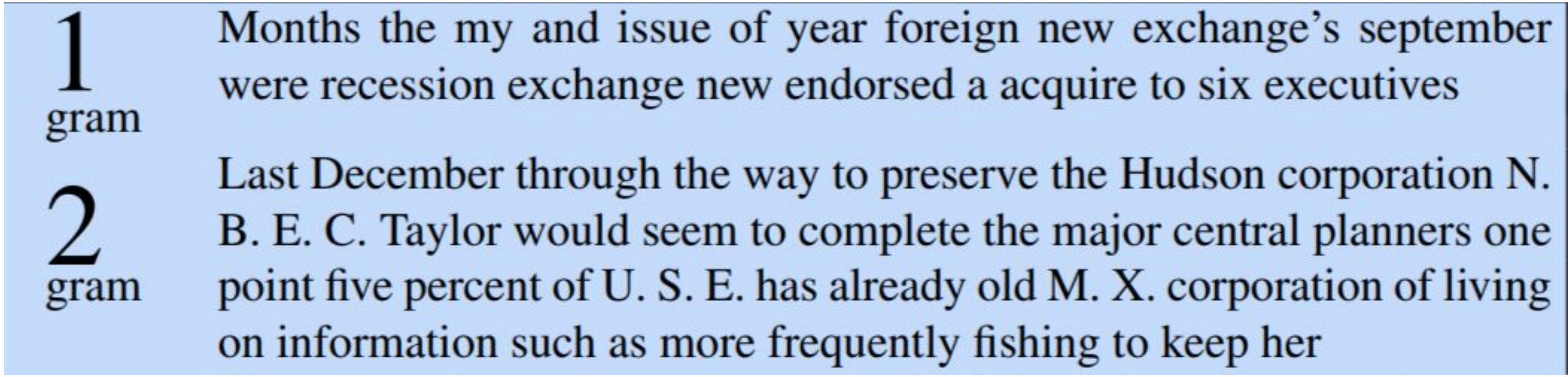
	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Sampling from a language model



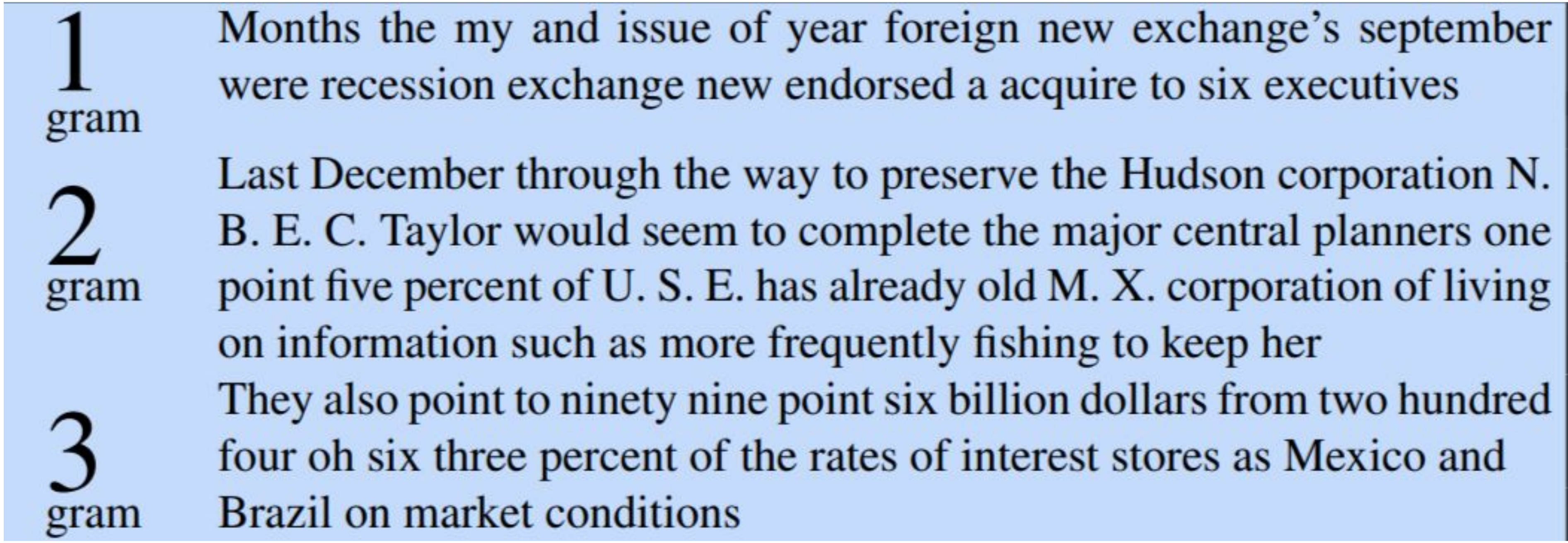
1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Sampling from a language model



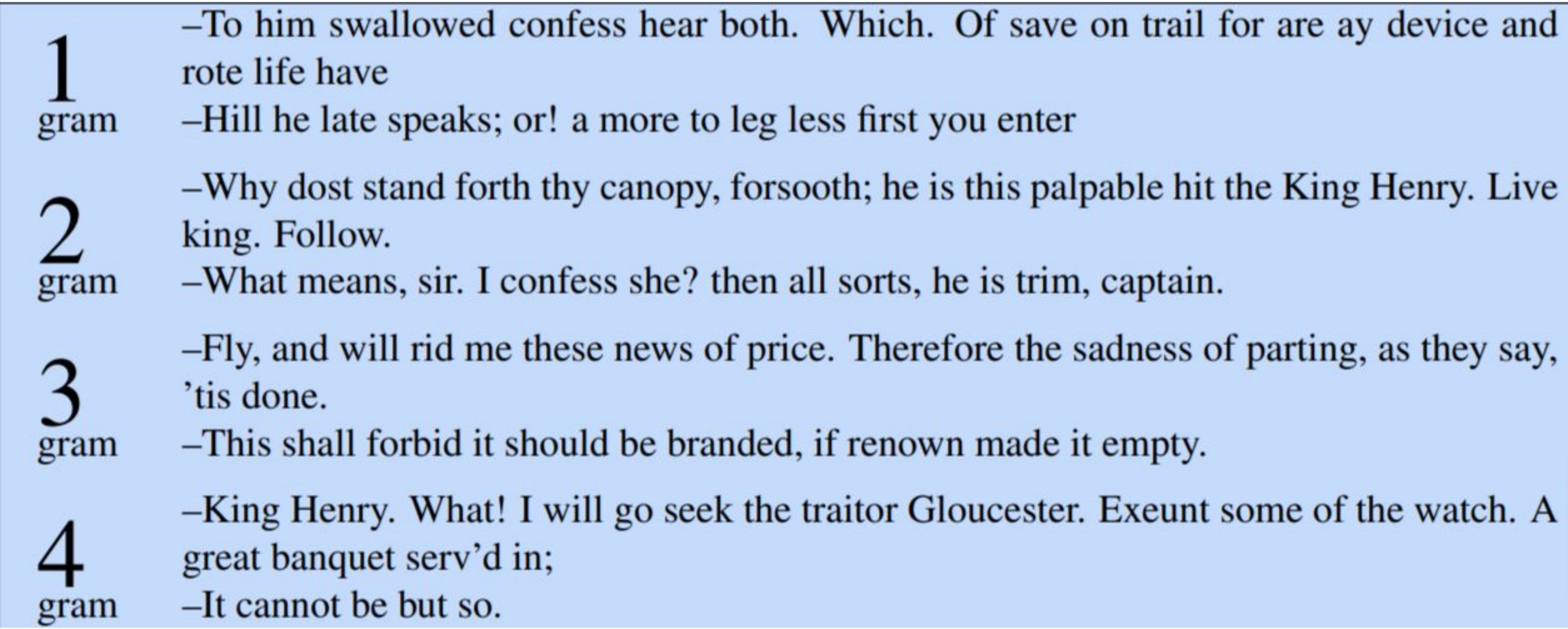
1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

Sampling from a language model



1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Sampling from a language model



1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, ’tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv’d in; –It cannot be but so.

Practical issues

- Multiplying very small numbers results in numerical underflow
  - we do every operation in log space
  - (also adding is faster than multiplying)

Markovian assumption is false

He is from France, so it makes sense that his first language is…

- We would want to model longer dependencies

Sparsity

- Maximum likelihood for estimating q
  - Let $c(w_1, \ldots, w_n)$ be the number of times that *n*-gram appears in a corpus

$$q(w_i \mid w_{i-2}, w_{i-1}) = \frac{c(w_{i-2}, w_{i-1}, w_i)}{c(w_{i-2}, w_{i-1})}$$

  - If vocabulary has 20,000 words ⇒ Number of parameters is 8×10^{12}!

Bias-variance tradeoff

- Given a corpus of length M

Trigram model:

$$q(w_i \mid w_{i-2}, w_{i-1}) = \frac{c(w_{i-2}, w_{i-1}, w_i)}{c(w_{i-1}, w_i)}$$

Bigram model:

$$q(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Unigram model:

$$q(w_i) = \frac{c(w_i)}{M}$$

Dealing with sparsity

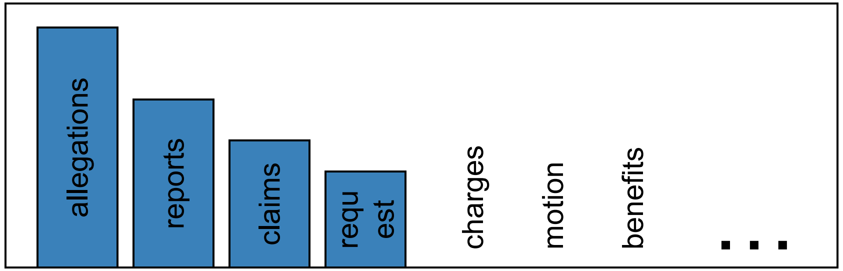
- For most N-grams, we have few observations
- General approach: modify observed counts to improve estimates
  - Back-off:
    - use trigram if you have good evidence;
    - otherwise bigram, otherwise unigram
  - Interpolation: approximate counts of N-gram using combination of estimates from related denser histories
  - Discounting: allocate probability mass for unobserved events by discounting counts for observed events

Discounting/smoothing methods

- We often want to make estimates from sparse statistics:

P(w | denied the)

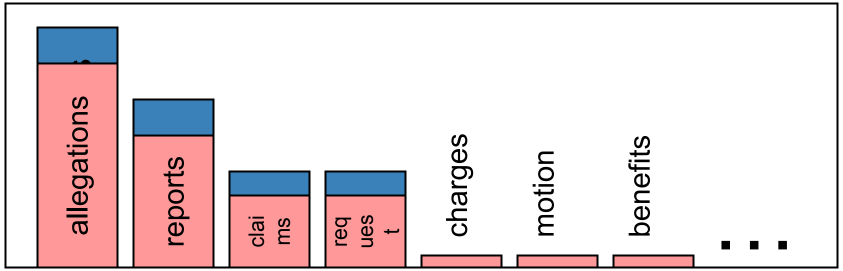
3	allegations
2	reports
1	claims
1	request
7	total



- Smoothing flattens spiky distributions so they generalize better:

P(w | denied the)

2.5	allegations
1.5	reports
0.5	claims
0.5	request
2	other
7	total



Linear interpolation

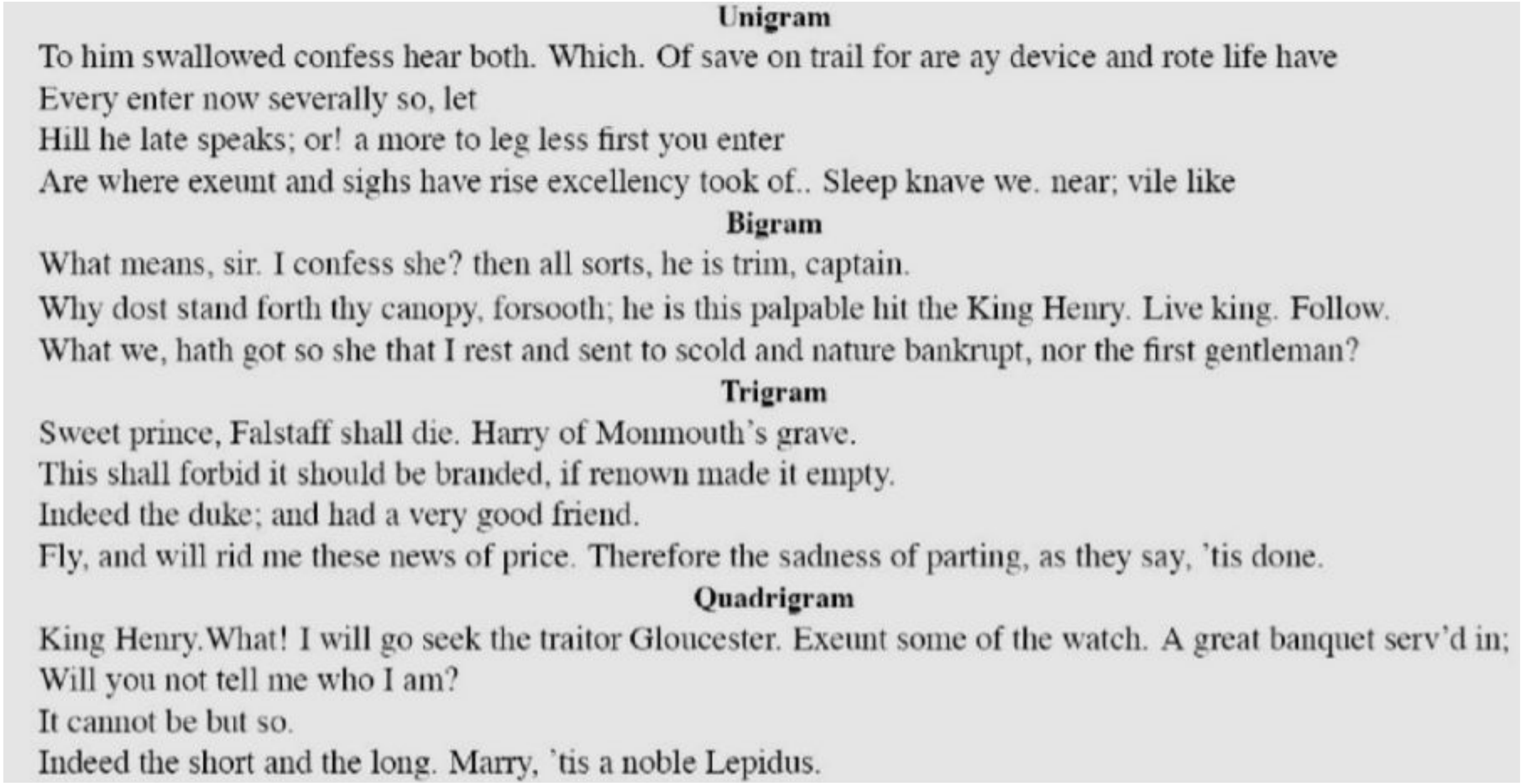
- Combine the three models to get all benefits

$$\begin{aligned} q_{LI}(w_i \mid w_{i-2}, w_{i-1}) = {} & \lambda_1 \times q(w_i \mid w_{i-2}, w_{i-1}) \\ & + \lambda_2 \times q(w_i \mid w_{i-1}) \\ & + \lambda_3 \times q(w_i) \end{aligned}$$

$$\lambda_i \geq 0,\ \ \lambda_1 + \lambda_2 + \lambda_3 = 1$$

Dealing with Out-of-vocabulary terms

- Define a special OOV or "unknown" symbol `<unk>`. Transform some (or all) rare words in the training data to `<unk>`
  - You cannot fairly compare two language models that apply different <unk> treatments
- Build a language model at the character level



Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have

Every enter now severally so, let

Hill he late speaks; or! a more to leg less first you enter

Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.

Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.

What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.

This shall forbid it should be branded, if renown made it empty.

Indeed the duke; and had a very good friend.

Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;

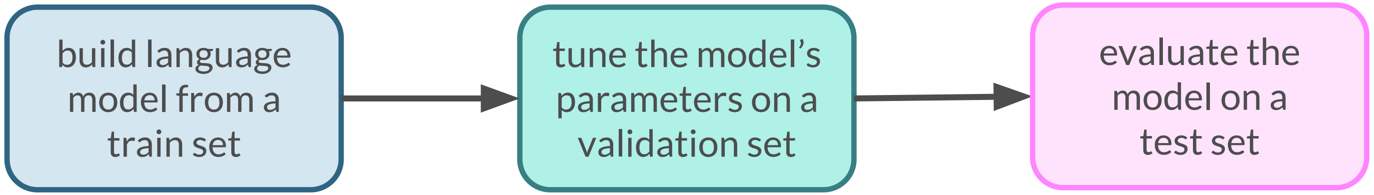
Will you not tell me who I am?

It cannot be but so.

Indeed the short and the long. Marry, 'tis a noble Lepidus.

Evaluation

- Extrinsic evaluation: build a new language model, use it for some task (MT, ASR, etc.)
- Intrinsic: measure how good we are at modeling language


build language model from a train set
tune the model's parameters on a validation set
evaluate the model on a test set

Extrinsic evaluation of N-gram models

- Best evaluation for comparing models A and B
  - Put each model in a task
    - spelling corrector, speech recognizer, MT system
  - Run the task, get an accuracy for A and for B
    - How many misspelled words corrected properly
    - How many words translated correctly
- Compare accuracy for A and B

Difficulty of extrinsic (in-vivo) evaluation of N-gram models

- Extrinsic evaluation
  - Time-consuming; can take days or weeks

So

- Sometimes use intrinsic evaluation: perplexity
  - Bad approximation
    - unless the test data looks just like the training data
  - So generally only useful in pilot experiments
  - But is helpful to think about

Evaluation: perplexity

- Test data: $S = \{s_1, s_2, \ldots, s_{sent}\}$
  - parameters are estimated on training data

$$p(\mathcal{S}) = \prod_{i=1}^{sent} p(s_i)$$

  - *sent* is the number of sentences in the test data

Evaluation: perplexity

- Test data: S = {s_1, s_2, …, s_{sent}}
  - parameters are estimated on training data

$$p(\mathcal{S}) = \prod_{i=1}^{sent} p(s_i)$$

$$\begin{aligned} p(\text{the dog barks STOP}) = & q(the \mid *, *)\times \\ & q(dog \mid *, the)\times \\ & q(barks \mid the, dog)\times \\ & q(STOP \mid dog, barks)\times \end{aligned}$$

  - *sent* is the number of sentences in the test data

Evaluation: perplexity

- Test data: $S = \{s_1, s_2, \ldots, s_{sent}\}$
  - parameters are estimated on training data

$$p(\mathcal{S}) = \prod_{i=1}^{sent} p(s_i)$$

$$\log_2 p(\mathcal{S}) = \sum_{i=1}^{sent} \log_2 p(s_i)$$

  - *sent* is the number of sentences in the test data

Evaluation: perplexity

- Test data: S = {s_1, s_2, …, s_{sent}}
  - parameters are estimated on training data

$$p(\mathcal{S}) = \prod_{i=1}^{sent} p(s_i)$$

$$\log_2 p(\mathcal{S}) = \sum_{i=1}^{sent} \log_2 p(s_i)$$

$$\text{perplexity} = 2^{-l},\ l = \frac{1}{M} \sum_{i=1}^{sent} \log_2 p(s_i)$$

  - *sent* is the number of sentences in the test data
  - M is the number of words in the test corpus

Evaluation: perplexity

- Test data: S = {s_1, s_2, …, s_{sent}}
    - parameters are estimated on training data

$$p(\mathcal{S}) = \prod_{i=1}^{sent} p(s_i)$$

$$\log_2 p(\mathcal{S}) = \sum_{i=1}^{sent} \log_2 p(s_i)$$

$$\text{perplexity} = 2^{-l}, \; l = \frac{1}{M} \sum_{i=1}^{sent} \log_2 p(s_i)$$

    - *sent* is the number of sentences in the test data
    - M is the number of words in the test corpus
    - A good language model has high p(S) and low perplexity

Language models

- Language models are distributions over sentences

$$P(w_1 \dots w_n)$$

- N-gram models are built from local conditional probabilities

$$P(w_1 \dots w_n) = \prod_i P(w_i | w_{i-k} \dots w_{i-1})$$

- The methods we've seen are backed by corpus n-gram counts

$$\hat{P}(w_i | w_{i-1}, w_{i-2}) = \frac{c(w_{i-2}, w_{i-1}, w_i)}{c(w_{i-2}, w_{i-1})}$$